

Intuitively, we think that two events should be independent if and only if the fact that one of them occurs does not affect the probability of the other one occurring.

The conditional probability of  $A$  given  $B$  is:

$$P(A | B) = \frac{P(A \cap B)}{P(B)}.$$

Events  $A$  and  $B$  are independent exactly when  $P(A)P(B) = P(A \cap B)$ , or equivalently,

$$P(A) = \frac{P(A \cap B)}{P(B)}$$

So now we see that  $P(A | B) = P(A)$  if and only if  $A$  and  $B$  are independent. This matches our intuition, namely, if we know that  $B$  occurs, it does not affect the probability of  $A$  occurring.