

Conditional Probabilities are actually Probabilities

1. $0 \leq P(A|B) \leq 1$

2. $P(S|B) = 1$

3. If A_j 's disjoint then $P(\bigcup_j A_j | B) = \sum_j P(A_j | B)$

Check: 1. $\frac{0 \leq P(A \cap B) \leq P(B)}{P(B)} \leq \frac{P(B)}{P(B)}$, divide throughout by $P(B)$, get
 $0 \leq P(A|B) \leq 1$

2. $P(S|B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

3. $P(\bigcup_j A_j | B) = \frac{P(\bigcup_j A_j \cap B)}{P(B)} = \frac{P(\bigcup_j (A_j \cap B))}{P(B)} = \sum_j \frac{P(A_j \cap B)}{P(B)} = \sum_j P(A_j | B)$