

Flip 5 fair coins in a row. Let B denote the event exactly 3 Heads.
 Let A denote event that flips 1 and 2 are both Heads.

Find $P(A|B)$

B : write down flips that are heads

1 2 3 } are in $A \cap B$

1 2 4 } are in $A \cap B$

1 2 5 } are in $A \cap B$

1 3 4 } are in $A^c \cap B$

1 3 5 } are in $A^c \cap B$

1 4 5 } are in $A^c \cap B$

2 3 4 } are in $A^c \cap B$

245 } are in $A^c \cap B$

2 3 5 } are in $A^c \cap B$

3 4 5 } are in $A^c \cap B$

$$P(A|B) = \frac{3}{10} \quad P(A^c|B) = \frac{7}{10}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{HHHTT}) + P(\text{HHTHT}) + P(\text{HHTTH})}{10 \cdot \left(\frac{1}{2}\right)^5}$$

$$= \frac{3 \cdot \left(\frac{1}{2}\right)^5}{10 \cdot \left(\frac{1}{2}\right)^5} = \frac{3}{10}$$