

Example of Bayes' Theorem with  $B$  decomposed into  $A \cap B$  or  $A^c \cap B$ . We know

$$P(A | B) = \frac{P(A)P(B | A)}{P(A)P(B | A) + P(A^c)P(B | A^c)}$$

Suppose we have two dice in a hat (one has 6 sides, and one has 20 sides). Pick one of the dice at random (each die is chosen with probability  $1/2$ ). If we obtain a "5" on the die when we roll it, what is the probability that the die had 20 sides?

Let event  $A$  denote the event that the 20-sided die was chosen. So  $A^c$  is the event that the 6-sided die was chosen. Let event  $B$  be the event that "5" appears on the chosen die when we roll it. (Notice we do not immediately have  $P(B)$ .)

$$P(A | B) = \frac{(1/2)(1/20)}{(1/2)(1/20) + (1/2)(1/6)} = 3/13 = .231$$

Know that  $P(A) = 1/2$  and  $P(A^c) = 1/2$  too. Note:  $P(B | A) = 1/20$ . Also note  $P(B | A^c) = 1/6$ .

In summary, if we get a "5" on the chosen die, it was the 20-sided die with probability  $P(A | B) = 3/13 = .231$ .

Automatically know the complementary probability (for free), i.e., the probability that the 6-sided die was chosen, given that a "5" appears, is  $P(A^c | B) = 10/13 = .769$ .