Bayes’ Theorem (third version)
Suppose we have several possible events of interest, say \( A_1, A_2, \ldots, A_n \). Suppose that they are disjoint and that their union is all of the sample space, i.e., \( \bigcup_j A_j = S \). In other words, the \( A_j \)’s form a partition of the sample space.
Now let’s calculate \( P(A_j \mid B) \).

\[
P(A_j \mid B) = \frac{P(A_j \cap B)}{P(B)}
\]

\[
= \frac{P(A_j \cap B)}{P(A_1 \cap B) + P(A_2 \cap B) + \ldots + P(A_n \cap B)}
\]

\[
= \frac{P(A_j)P(B \mid A_j)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \ldots + P(A_n)P(B \mid A_n)}
\]