

Bayes' Theorem (third version)

Suppose we have several possible events of interest, say A_1, A_2, \dots, A_n . Suppose that they are disjoint and that their union is all of the sample space, i.e., $\bigcup_j A_j = S$. In other words, the A_j 's form a partition of the sample space.

Now let's calculate $P(A_j | B)$.

$$\begin{aligned} P(A_j | B) &= \frac{P(A_j \cap B)}{P(B)} \\ &= \frac{P(A_j \cap B)}{P(A_1 \cap B) + P(A_2 \cap B) + \dots + P(A_n \cap B)} \\ &= \frac{P(A_j)P(B | A_j)}{P(A_1)P(B | A_1) + P(A_2)P(B | A_2) + \dots + P(A_n)P(B | A_n)} \end{aligned}$$