

Use conditional probabilities to find the intersection of a collection of events. In other words, consider events  $A_1, A_2, \dots, A_n$ , and find the probability that all  $n$  of the events occur.

We want  $P(A_1 \cap A_2 \cap \dots \cap A_n)$ .

We do this by finding the probability  $A_1$  occurs, times the probability  $A_2$  occurs given that  $A_1$  occurred, times the probability  $A_3$  occurs given that  $A_1$  and  $A_2$  occurred, etc., etc.

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \cdots P(A_n|A_1 \cap A_2 \cap \dots \cap A_{n-1}).$$

Example: Consider three children who each pick 1 cookie from a cookie jar, without replacement. Suppose that the cookie jar has 12 cookies, exactly 5 of which are chocolate. What is the probability that all three children get a chocolate cookie?

Let  $A_j$  be the event that the  $j$ th child gets a chocolate cookie.

We want  $P(A_1 \cap A_2 \cap A_3)$ .

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2 | A_1)P(A_3 | A_1 \cap A_2) = (5/12)(4/11)(3/10) = 1/22 = 0.045 \dots$$