

Flip a coin until 1st head appears.

Let X be the # of flips until (and including) the first head.

$$S = \{H, TH, TTH, TTTH, \dots, \underbrace{TTT \dots T}_j H, \dots, TTTT \dots\}$$

prob's $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \dots \quad \downarrow \quad \downarrow \quad \downarrow$

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \dots \quad \frac{1}{2^{j-1}} \quad \frac{1}{2^j} \quad \dots$$

Check: $\sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^j = 1 \quad \checkmark$

$$P(X=1) = P(\{H\}) = \frac{1}{2}$$

$$P(X=2) = P(\{TH\}) = \frac{1}{4}$$

$$\vdots$$

$$P(X=j) = P(\{\underbrace{T \dots T}_{j-1} H\}) = \left(\frac{1}{2}\right)^j$$

$$P(X \leq 3) = P(\{H, TH, TTH\}) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$$

$$P(X > 3) = P(S \setminus \{H, TH, TTH\}) = 1 - P(X \leq 3) = 1 - \frac{7}{8} = \frac{1}{8}$$

\uparrow set minus $S \setminus \{H, TH, TTH\}^c$

\downarrow
 Only way $X > 3$
 is if first 3 flips are tails $P(X > 3) = \frac{1}{8}$

$$P(2 \leq X \leq 5) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$$

$$= \frac{8 + 4 + 2 + 1}{32}$$

$$= \frac{15}{32}$$