

Joint probability mass function of two random variables, also known as the joint PMF or the joint mass. The joint mass of two random variables gives the probability that each of the random variables takes on the specified values. For example, $p_{X,Y}(3, 10) = P(X = 3, Y = 10)$. At the root level,

$$p_{X,Y}(3, 10) = P(\{\omega \mid X(\omega) = 3, Y(\omega) = 10\})$$

In general, $p_{X,Y}(x, y) = P(X = x, Y = y)$.

Similarly, we can define the joint cumulative distribution function of two random variables, also called the joint CDF. For example, we want $F_{X,Y}(2, 17) = P(X \leq 2, Y \leq 17)$. Again, this all comes back to outcomes and events,

$$F_{X,Y}(2, 17) = P(\{\omega \mid X(\omega) \leq 2, Y(\omega) \leq 17\})$$

In general, define $F_{X,Y}(x, y) = P(X \leq x, Y \leq y)$.