

Example: Consider four children who are born independently (say) from four different mothers. Let  $X$  be the number of boys among the four children, and let  $Y$  be the number of girls. Notice: always  $X + Y = 4$ .

The joint mass in this case is

$$p_{X,Y}(0,4) = 1/16$$

$$p_{X,Y}(1,3) = 4/16$$

$$p_{X,Y}(2,2) = 6/16$$

$$p_{X,Y}(3,1) = 4/16$$

$$p_{X,Y}(4,0) = 1/16$$

Check, for instance, does  $\sum_x \sum_y p_{X,Y}(x,y) = 1$ ? Yes. Note also that  $p_{X,Y}(x,y) = 0$  otherwise, i.e., except for these five values, the joint mass is 0.

What about the joint CDF? For instance, what is  $F_{X,Y}(2,3)$ ? That is the probability  $X \leq 2$  and  $Y \leq 3$ . So

$$F_{X,Y}(2,3) = P(X = 1, Y = 3) + P(X = 2, Y = 2) = 4/16 + 6/16 = 10/16 = 5/8.$$