

Example: Consider four children who are born independently (say) from four different mothers. Let X be the number of boys among the four children, and let Y be the number of girls. Notice: always $X + Y = 4$.

The joint mass in this case is

$$p_{X,Y}(0,4) = 1/16$$

$$p_{X,Y}(1,3) = 4/16$$

$$p_{X,Y}(2,2) = 6/16$$

$$p_{X,Y}(3,1) = 4/16$$

$$p_{X,Y}(4,0) = 1/16$$

Check, for instance, does $\sum_x \sum_y p_{X,Y}(x,y) = 1$? Yes. Note also that $p_{X,Y}(x,y) = 0$ otherwise, i.e., except for these five values, the joint mass is 0.

What about the joint CDF? For instance, what is $F_{X,Y}(2,3)$? That is the probability $X \leq 2$ and $Y \leq 3$. So

$$F_{X,Y}(2,3) = P(X = 1, Y = 3) + P(X = 2, Y = 2) = 4/16 + 6/16 = 10/16 = 5/8.$$