

Using the joint mass to calculate the mass of a single random variable. For instance, consider two random variables X and Y . If we sum the values of the joint mass $p_{X,Y}(x, y)$ over all possible y values,

$$\sum_y p_{X,Y}(x, y) = \sum_y P(X = x, Y = y) = P(X = x, Y = \text{anything}) = P(X = x) = p_X(x)$$

In summary, if I want the mass of X itself, I can sum the joint mass of X and Y , over all possible values of Y , and in a sense, the Y is summed out of the picture:

$$p_X(x) = \sum_y p_{X,Y}(x, y)$$

Similarly, if you have a joint CDF, and you want to get the single variable CDF, for instance, the CDF of just X by itself, we can do it:

$$\lim_{y \rightarrow \infty} F_{X,Y}(x, y) = \lim_{y \rightarrow \infty} P(X \leq x, Y \leq y) = P(X \leq x, Y \text{ is anything}) = P(X \leq x) = F_X(x)$$

So in summary, to get the single variable CDF, say, of X , you can take the joint CDF of X and Y , and take the limit of the y part as $y \rightarrow \infty$. I.e.,

$$F_X(x) = \lim_{y \rightarrow \infty} F_{X,Y}(x, y)$$