

Consider two random variables X and Y that are indicator random variables for (respectively) events A and B . Recall that this means $X=1 \leftrightarrow A$ occurs, $X=0$ otherwise. $Y=1 \leftrightarrow B$ occurs, $Y=0$ otherwise.

(Idea: A and B are independent events if and only if X, Y independent random variables.

$P_{X,Y}(1,1) = P(A \cap B)$ $P_X(1)P_Y(1) = P(A)P(B)$	$\begin{cases} \text{equal} \\ \text{if and only if} \\ A, B \text{ indep.} \end{cases}$	$P_{X,Y}(1,0) = P(A \cap B^c)$ $P_X(1)P_Y(0) = P(A)P(B^c)$	$\begin{cases} \text{equal if and only if} \\ A, B \text{ independent} \end{cases}$
$P_{X,Y}(0,0) = P(A^c \cap B^c)$ $P_X(0)P_Y(0) = P(A^c)P(B^c)$	$\begin{cases} \text{equal iff} \\ A, B \text{ indep} \end{cases}$	$P_{X,Y}(0,1) = P(A^c \cap B)$ $P_X(0)P_Y(1) = P(A^c)P(B)$	$\begin{cases} \text{equal iff } A, B \text{ indep.} \end{cases}$

So in summary, random variables X, Y are independent exactly when events A, B are independent.