Joint mass of a collection of random variables, $X_1, X_2, ..., X_n$

$$p_{X_1, ..., X_n}(x_1, ..., x_n) = P(X_1 = x_1, X_2 = x_2, ..., X_n = x_n)$$

Joint CDF of a collection of random variables $X_1, X_2, ..., X_n$

$$F_{X_1, ..., X_n}(x_1, ..., x_n) = P(X_1 \leq x_1, X_2 \leq x_2, ..., X_n \leq x_n)$$

Independence of the collection $X_1, ..., X_n$ is equivalent to either of these conditions (which are equivalent to each other)

$$p_{X_1, ..., X_n}(x_1, ..., x_n) = p_{X_1}(x_1) \cdots p_{X_n}(x_n) \text{ must hold for all } x_1, ..., x_n$$

or equivalently

$$F_{X_1, ..., X_n}(x_1, ..., x_n) = F_{X_1}(x_1) \cdots F_{X_n}(x_n) \text{ again must hold for all } x_1, ..., x_n$$

Similarly, if $A_1, ..., A_n$ are some events and if $X_1, ..., X_n$ are indicator random variables for those events, i.e.

$$X_j = 1 \iff A_j \text{ occurs}$$

Then $X_1, ..., X_n$ independent if and only if

$$X_j = 0 \text{ otherwise} \quad \text{the events } A_1, ..., A_n \text{ are independent.}$$

Remember from studying events that $P(A \cap B) = P(A)P(B|A)$

Similarly we can write an equation for the joint mass of $X$ and $Y$ in terms of (say) the mass of $X$ and the conditional mass of $Y$ given $X$:

$$p_{X,Y}(x,y) = p(X=x, Y=y) = p(X=x)p(Y=y|X=x) = p_X(x)p_{Y|X}(y|x)$$

In summary:

$$p_{X,Y}(x,y) = p_X(x)p_{Y|X}(y|x).$$

Hint: When working towards independence of random variables, think about the analogous situation with events and try to do something similar. Often such a strategy will work.