

Example of calculating the expected value of a discrete random variable
 Let X be the number of girls born in 4 independent births of babies.
 Calculate

$$\begin{aligned}
 E(X) &= \sum_{\omega} X(\omega)P(\{\omega\}) \\
 &= 0P(\{(b, b, b, b)\}) \\
 &\quad + 1P(\{g, b, b, b\}) + 1P(\{b, g, b, b\}) + 1P(\{b, b, g, b\}) + 1P(\{b, b, b, g\}) \\
 &\quad + 2P(\{g, g, b, b\}) + 2P(\{g, b, g, b\}) + 2P(\{g, b, b, g\}) \\
 &\quad\quad + 2P(\{b, g, g, b\}) + 2P(\{b, g, b, g\}) + 2P(\{b, b, g, g\}) \\
 &\quad + 3P(\{g, g, g, b\}) + 3P(\{g, g, b, g\}) + 3P(\{g, b, g, g\}) + 3P(\{b, g, g, g\}) \\
 &\quad + 4P(\{g, g, g, g\}) \\
 &= 0(1/16) + 1(4/16) + 2(6/16) + 3(4/16) + 4(1/16) \\
 &= \sum_j x_j P(X = x_j)
 \end{aligned}$$

In either case, we see that

$$E(X) = \frac{(0)(1) + (1)(4) + (2)(6) + (3)(4) + (4)(1)}{16} = 32/16 = 2$$