

Example where X is the number of girls in four births of babies.

$$E(X) = (0)\left(\frac{1}{16}\right) + (1)\left(\frac{4}{16}\right) + (2)\left(\frac{6}{16}\right) + (3)\left(\frac{4}{16}\right) + (4)\left(\frac{1}{16}\right)$$

Notice that $E(X)$ is a weighted sum of the possible values that X takes on, here are 0, 1, 2, 3, 4. The weights themselves are probabilities, here $\frac{1}{16}$, $\frac{4}{16}$, $\frac{6}{16}$, $\frac{4}{16}$, $\frac{1}{16}$. Notice also that the weights must sum to 1 because we have a partition of the sample space. Check: Circle/underline the various weights, make sure that they sum to 1 altogether. If they do not sum to 1, there is some fundamental problem.

No reason that $E(X)$ has to be (itself) one of the values X takes on. For instance, roll a die, let X be the result,

$$\begin{aligned} E(X) &= (1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (3)\left(\frac{1}{6}\right) + (4)\left(\frac{1}{6}\right) + (5)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{6}\right) \\ &= \frac{1}{6}(1+2+3+4+5+6) \\ &= \frac{21}{6} = 3.5 \leftarrow \text{not a value } X \text{ itself can take on.} \end{aligned}$$

That's OK! Do not just round to 3 or 4 for safety.