

Example: Expectation. Let X be the number of draws required to find a certain item among n items total. Assume that the items are not replaced after they are chosen. Also assume that the selections are done blindly.



$X=1$ with probability $\frac{1}{n}$
 $X=2$ with probability $\frac{1}{n}$
 $X=3$ with probability $\frac{1}{n}$

think: all n items equally likely to be 2nd

.....
 $X=n$ with probability $\frac{1}{n}$

$$\begin{aligned}
 E(X) &= (1)\left(\frac{1}{n}\right) + (2)\left(\frac{1}{n}\right) + (3)\left(\frac{1}{n}\right) + \dots + (n)\left(\frac{1}{n}\right) \\
 &= \left(\frac{1}{n}\right)(1 + 2 + 3 + \dots + n) \\
 &= \left(\frac{1}{n}\right)\left(\frac{n(n+1)}{2}\right) \\
 &= \frac{n+1}{2}.
 \end{aligned}$$

Check: $(n)\left(\frac{1}{n}\right) = 1$
 weights sum to 1.