Example: Let $X$ denote the number of girls born in the births of 4 babies, i.e. how many are girls? 

Idea: Define $X_j = 1$ if $j$th baby is a girl 

0 otherwise

Notice: Always $X = X_1 + X_2 + X_3 + X_4$. It is just like counting on your fingers. E.g. if only babies 2 and 3 are girls, then

$X_1 = 0$, $X_2 = 1$, $X_3 = 1$, $X_4 = 0$  

$X = 0 + 1 + 1 + 0 = 2$

Notice $E(X_j) = P(A_j)$ where $A_j$ is the event that the $j$th baby is a girl.

So $E(X_j) = \frac{1}{2}$ for each $j$.

So altogether $E(X) = E(X_1 + X_2 + X_3 + X_4)$

$= E(X_1) + E(X_2) + E(X_3) + E(X_4) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$.

Same expected value we got as when we first treated the problem. Also: notice we never computed the mass of $X$,

i.e. we never wrote down the numbers $\frac{1}{16}$, $\frac{4}{16}$, $\frac{6}{16}$, $\frac{4}{16}$, $\frac{1}{16}$.

So this method is easier, more straightforward, and more natural since we are used to counting occurrences (yes/no) on your fingers!