One more nice idea that works with random variables that are nonnegative and integer valued, i.e. works if X takes on only (some) of the values $0, 1, 2, 3, 4, \ldots\ldots$

$\cancel{0P(X=0)}$??? $= 0$!

$$E(X) = 1P(X=1) + 2P(X=2) + 3P(X=3) + 4P(X=4) + 5P(X=5) + \cdots\cdots$$

$$\begin{aligned} = \; & P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + \cdots\cdots \\ & + P(X=2) + P(X=3) + P(X=4) + P(X=5) + \cdots\cdots \\ & + P(X=3) + P(X=4) + P(X=5) \\ & + P(X=4) + P(X=5) \\ & + P(X=5) \end{aligned}$$

$$= P(X\geq 1) + P(X \geq 2) + P(X\geq 3) + P(X\geq 4) + P(X\geq 5) + P(X \geq 6) + \cdots\cdots$$

$$\boxed{E(X) = \sum_{j=1}^{\infty} P(X \geq j)}$$

Notice a couple things different from the regular statement of expected values.

no leading-order term out front of the probability

we do <u>not</u> put the mass of X here, but rather, the probability that $X \geq$ some value. e.g. 3rd term is $P(X\geq 3)$.

Again we see that reorganizing terms can lead to neat simplifications.