Example: Say we consider the births of 4 children, from separate mothers. Let X denote the # of girls that are born. Let $h(X) = X^2$. Find $E[h(X)]$.

Two methods: (1) Use the individual outcomes

$$
\begin{aligned}
E[h(X)] = {} & 0^2 P(\{(b,b,b,b)\}) \\
& + 1^2 P(\{(g,b,b,b)\}) + 1^2 P(\{(b,g,b,b)\}) + 1^2 P(\{(b,b,g,b)\}) \\
& \quad + 1^2 P(\{(b,b,b,g)\}) \\
& + 2^2 P(\{(g,g,b,b)\}) + 2^2 P(\{(g,b,g,b)\}) + 2^2 P(\{(g,b,b,g)\}) \\
& \quad + 2^2 P(\{(b,g,g,b)\}) + 2^2 P(\{(b,g,b,g)\}) + 2^2 P(\{(b,b,g,g)\}) \\
& + 3^2 P(\{(g,g,g,b)\}) + 3^2 P(\{(g,g,b,g)\}) + 3^2 P(\{(g,b,g,g)\}) \\
& \quad + 3^2 P(\{(b,g,g,g)\}) \\
& + 4^2 P(\{(g,g,g,g)\})
\end{aligned}
$$

$$
\begin{aligned}
= {} & 0^2 \cdot \tfrac{1}{16} + \left(1^2 \cdot \tfrac{1}{16} + 1^2 \cdot \tfrac{1}{16} + 1^2 \cdot \tfrac{1}{16} + 1^2 \cdot \tfrac{1}{16}\right) \\
& + \left(2^2 \cdot \tfrac{1}{16} + 2^2 \tfrac{1}{16} + 2^2 \cdot \tfrac{1}{16} + 2^2 \cdot \tfrac{1}{16} + 2^2 \cdot \tfrac{1}{16} + 2^2 \cdot \tfrac{1}{16}\right) \\
& + \left(3^2 \cdot \tfrac{1}{16} + 3^2 \cdot \tfrac{1}{16} + 3^2 \cdot \tfrac{1}{16} + 3^2 \cdot \tfrac{1}{16}\right) \\
& + 4^2 \cdot \tfrac{1}{16}
\end{aligned}
$$

$$= \frac{(0)(1) + (4)(1) + (6)(4) + (4)(9) + (1)(16)}{16}$$

$$= \frac{80}{16} = \boxed{5}$$

Method #2 Group by value of X

$P(X=0) = \frac{1}{16}$ $\quad P(X=1) = \frac{4}{16}$ $\quad P(X=2) = \frac{6}{16}$ $\quad P(X=3) = \frac{4}{16}$ $\quad P(X=4) = \frac{1}{16}$

$$E[h(X)] = (0^2)\left(\tfrac{1}{16}\right) + (1^2)\left(\tfrac{4}{16}\right) + (2^2)\left(\tfrac{6}{16}\right) + (3^2)\left(\tfrac{4}{16}\right) + (4^2)\left(\tfrac{1}{16}\right)$$

(Probability weights sum to 1. ✓)

$$= \frac{0 + 4 + 24 + 36 + 16}{16} = \frac{80}{16} = 5$$