More nice facts about the variance and the expected value of independent random variables.

If X, Y independent and g, h are real valued functions then

$$E(g(X) \cdot h(Y)) = E(g(X)) \cdot E(h(Y))$$

Why? Informally, write $U = g(X)$, $V = h(Y)$ and note U, V independent since X, Y are independent.

all we need to show is the simpler version:

If U, V are independent random variables,

$$E(U \cdot V) = E(U) \cdot E(V).$$

Another nice fact: If $X_1, X_2, \ldots, X_n$ are independent random variables

and if $a_1, a_2, \ldots, a_n$ are constants

then $Var(a_1 X_1 + a_2 X_2 + \ldots + a_n X_n) = a_1^2 Var(X_1) + a_2^2 Var(X_2) + \ldots + a_n^2 Var(X_n)$

In particular, using $a_1 = a_2 = \ldots = a_n = 1$ if $X_1, X_2, \ldots, X_n$ are independent then $Var(X_1 + X_2 + \ldots + X_n) = Var(X_1) + Var(X_2) + \ldots + Var(X_n)$

In particular, if $Var(X_1) = Var(X_2) = \ldots = Var(X_n)$

this simplifies to

$$Var(X_1 + X_2 + \ldots + X_n) = n Var(X_1)$$

(again need independence of $X_1, X_2, \ldots, X_n$ to use this).