

Bernoulli random variables, also called indicator random variables.

Take on only two values, namely, 0 or 1. We say X is Bernoulli(p) random variable if $X = 1$ with probability p and $X = 0$ with probability $q = 1 - p$.

Expected value

$$E(X) = 1P(X = 1) + 0P(X = 0) = P(X = 1) = p$$

Variance

$$\text{Var}(X) = E(X^2) - (E(X))^2 = p - p^2 = p(1 - p) = pq.$$

Note that $X^2 = X$ always (when X is Bernoulli). Why? If $X = 1$ then $X^2 = 1$ too. If $X = 0$ then $X^2 = 0$ too. So $E(X^2) = E(X) = p$.

In general, if X is Bernoulli, then $X^j = X$ for any positive integer j . Why? If $X = 1$ then $X^j = 1$ too. If $X = 0$ then $X^j = 0$ too.