Binomial coefficients: \( \binom{n}{j} \) read as "n choose j" defined as \( \frac{n!}{j!(n-j)!} \)

This is # of ways to pick exactly j out n items in a row, without regard to the order of picking i.e. without noting order of selection.

\[ \binom{5}{3} \]

5 possibilities

with the order of selection noted, there are \( \binom{5}{4}\times 3! \) ways to pick.

4 remaining possibilities

3 remaining possibilities

\[ \binom{5}{4}\times 3! = \frac{5!}{2!} = 60 \]

Also there are 3! ways this triple could have been picked, so I overcounted by a factor of 3! if I want to ignore the order of selection.

2nd, 3rd, 5th items

So there are really only \( \frac{5!}{3!2!} = 10 \) ways if we ignore order of selection.

10 ways: 1, 2, 3
1, 2, 4
1, 2, 5
1, 3, 4
1, 3, 5
1, 4, 5
2, 3, 4
2, 3, 5
2, 4, 5
3, 4, 5

Without regarding the order of selection, e.g. without colouring them as you pick them.

So if we have 5 trials, and we want exactly 3 successes, there are

\[ \binom{5}{3} = \frac{5!}{3!2!} = 10 \] ways that this could happen.

Binomial coefficients play a key role in defining Binomial random variables.