

Expected value and variance of Binomial random variables

Perhaps the easiest way to compute the expected value of a Binomial random variable is to use the interpretation that a Binomial(n, p) random variable is the sum of n independent Bernoulli(p) random variables. E.g., if X is a Binomial(n, p) random variable, we can think of

$$X = X_1 + X_2 + \dots + X_n$$

where the X_i 's are independent Bernoulli(p) random variables. Each X_i has expected value p , so

$$E(X) = E(X_1 + \dots + X_n) = E(X_1) + \dots + E(X_n) = p + \dots + p = np.$$

For the variance, **because the X_i 's are independent**, the variance of the sum is equal to the sum of the variance. So

$$\text{Var}(X) = \text{Var}(X_1 + \dots + X_n) = \text{Var}(X_1) + \dots + \text{Var}(X_n) = pq + \dots pq = npq.$$