Earlier example: Recall the situation where there are 4 births from 4 mothers (e.g., no twins), and let X denote the number of girls born among the 4 children. Notice that X is a Binomial$(4, 1/2)$ random variable, i.e., $n = 4$ and $p = 1/2$.

We can see the use of the Binomial coefficients in this light. Let's recompute the mass of X:

$$p_X(0) = P(X = 0) = \binom{4}{0}(1/2)^0(1/2)^4 = (1)(1/16) = 1/16 \qquad \text{because } \tbinom{4}{0} = \tfrac{4!}{0!4!} = 1$$

$$p_X(1) = P(X = 1) = \binom{4}{1}(1/2)^1(1/2)^3 = (4)(1/16) = 1/4 \qquad \text{because } \tbinom{4}{1} = \tfrac{4!}{1!3!} = 4$$

$$p_X(2) = P(X = 2) = \binom{4}{2}(1/2)^2(1/2)^2 = (6)(1/16) = 3/8 \qquad \text{because } \tbinom{4}{2} = \tfrac{4!}{2!2!} = 6$$

$$p_X(3) = P(X = 3) = \binom{4}{3}(1/2)^3(1/2)^1 = (4)(1/16) = 1/4 \qquad \text{because } \tbinom{4}{3} = \tfrac{4!}{3!1!} = 4$$

$$p_X(4) = P(X = 4) = \binom{4}{4}(1/2)^4(1/2)^0 = (1)(1/16) = 1/16 \qquad \text{because } \tbinom{4}{4} = \tfrac{4!}{4!0!} = 1$$

We note a few things: The mass adds up to 1, as it should $1/16+1/4+3/8+1/4+1/16 = 1$.

Also note that $\binom{n}{j} = \binom{n}{n-j}$. Why? $\binom{n}{j} = \frac{n!}{j!(n-j)!} = \frac{n!}{(n-j)!j!} = \binom{n}{n-j}$. Intuitively this makes sense, because if we have n items, and we choose j of them, we have avoided exactly $n - j$ items. So we could just switch your view, and (instead) decided which items to avoid (instead of which items to choose), and this is $\binom{n}{n-j}$. For instance, $\binom{4}{1} = \binom{4}{3}$.