Earlier example: Recall the situation where there are 4 births from 4 mothers (e.g., no twins), and let $X$ denote the number of girls born among the 4 children. Notice that $X$ is a $\text{Binomial}(4, \frac{1}{2})$ random variable, i.e., $n = 4$ and $p = 1/2$.

We can see the use of the Binomial coefficients in this light. Let’s recompute the mass of $X$:

$$p_X(0) = P(X = 0) = \binom{4}{0}(1/2)^0(1/2)^4 = (1)(1/16) = 1/16$$  because $(4)_0 = \frac{4!}{0!4!} = 1$

$$p_X(1) = P(X = 1) = \binom{4}{1}(1/2)^1(1/2)^3 = (4)(1/16) = 1/4$$  because $(4)_1 = \frac{4!}{1!3!} = 4$

$$p_X(2) = P(X = 2) = \binom{4}{2}(1/2)^2(1/2)^2 = (6)(1/16) = 3/8$$  because $(4)_2 = \frac{4!}{2!2!} = 6$

$$p_X(3) = P(X = 3) = \binom{4}{3}(1/2)^3(1/2)^1 = (4)(1/16) = 1/4$$  because $(4)_3 = \frac{4!}{3!1!} = 4$

$$p_X(4) = P(X = 4) = \binom{4}{4}(1/2)^4(1/2)^0 = (1)(1/16) = 1/16$$  because $(4)_4 = \frac{4!}{4!0!} = 1$

We note a few things: The mass adds up to 1, as it should $1/16 + 1/4 + 3/8 + 1/4 + 1/16 = 1$.

Also note that $(n)_j = \binom{n}{n-j}$. Why? $(n)_j = \frac{n!}{j!(n-j)!} = \frac{n!}{(n-j)!} = \binom{n}{n-j}$. Intuitively this makes sense, because if we have $n$ items, and we choose $j$ of them, we have avoided exactly $n - j$ items. So we could just switch your view, and (instead) decided which items to avoid (instead of which items to choose), and this is $\binom{n}{n-j}$. For instance, $(4)_1 = \binom{4}{3}$. 