Geometric random variables

There are two kinds of geometric random variables, either (1) number of trials needed until the first success, and including the first success itself, or (2) number of trials that fail before (strictly before) the first success occurs. These are different random variables, but you might see both of them in the literature, etc. We call the first type "Geometric" and we call the second type "Geometric number of losses". Notice that if X is Geometric, then $X - 1$ is a Geometric number of losses, because with $X - 1$ we just do not count the success itself.

Example: Suppose that we have an infinite sequence of independent trials, each of which succeeds with probability p. (The p is common across all trials.) For instance, if we have "fail, fail, fail, fail, fail, fail, success", we say that a Geometric random variable X takes on value 7 in this case, i.e., 7 trials were needed to see the first success. Here, $X - 1 = 6$, so we could (instead) say that a Geometric number of losses here is equal to 6. We usually look at the first interpretation.

If X is Geometric with probability of success p on each trial, then the mass of X is

$$p_X(x) = q^{x-1}p \qquad \text{where } p \text{ is the probability of success on a trial, and } q = 1 - p, \text{ here } x \geq 1.$$

In that case, we say X is a Geometric(p) random variable. The possible values X takes on are $1, 2, 3, \ldots$. Can't take on value 0, because you need at least one trial to get a success.

Another way to think is: Let $X_1, X_2, \ldots$ be independent trials, each with probability of success p. Then X is Geometric(p) if X equals the first index M so that $X_M = 1$. (Necessarily $X_1, X_2, \ldots, X_{M-1} = 0$.) For instance, our example above, $M = 7$, because $X_1 = X_2 = \ldots = X_6 = 0$ and $X_7 = 1$.

For comparison, if (say) Y is a Geometric number of losses random variable, then the mass of Y is

$$p_Y(y) = q^y p$$

where $q = 1 - p$, and this holds for $y = 0, 1, 2, \ldots$. If is possible to have $Y = 0$ because we might get a success right off the bat, and therefore 0 losses are needed until the first success.