

# Negative binomial random variables

What they are NOT is the negative of a Binomial  $(n, p)$  !!

Instead, they are a generalization of Geometric  $(p)$  random variables.

Idea: Geometric  $(p)$

$$\underbrace{F F F F F S}_{X = 7}$$

Where  $X$  is our Geometric  $(p)$  random variable i.e. number of trials until 1st success

$$\underbrace{F F F S}_{\text{1st Geom}} \mid \underbrace{F F F F S}_{\text{2nd Geom}} \mid \underbrace{F S}_{\text{3rd Geom}} \mid \underbrace{F F F F S}_{\text{4th Geom}}$$

the total number of flips until the 4th success, assuming each trial has probability of success  $p$ , and independent trials, is a Negative Binomial random variable with parameters  $r=4$  and  $p$ .

In general, a Negative Binomial  $(r, p)$  random variable is the number of trials needed until the  $r$ th success occurs.

Equivalently, a Negative Binomial  $(r, p)$  random variable is the sum of  $r$  independent Geometric  $(p)$  random variables

$$\underbrace{F F F S}_{X_1} \mid \underbrace{F F F F S}_{X_2} \mid \underbrace{F S}_{X_3} \mid \underbrace{F F F F S}_{X_4}$$

$$X = 18 = X_1 + X_2 + X_3 + X_4 = 4 + 6 + 2 + 6$$