

Negative binomial random variables

What they are NOT is the negative of a Binomial (n, p) !!

Instead, they are a generalization of Geometric (p) random variables.

Idea: Geometric (p)

$$\underbrace{F F F F F S}_{X = 7}$$

Where X is our Geometric (p) random variable i.e. number of trials until 1st success

$$\underbrace{F F F S}_{1^{st} \text{ Geom}} \mid \underbrace{F F F F S}_{2^{nd} \text{ Geom}} \mid \underbrace{F S}_{3^{rd} \text{ Geom}} \mid \underbrace{F F F F S}_{4^{th} \text{ Geom}}$$

the total number of flips until the 4th success, assuming each trial has probability of success p , and independent trials, is a Negative Binomial random variable with parameters $r=4$ and p .

In general, a Negative Binomial (r, p) random variable is the number of trials needed until the r th success occurs.

Equivalently, a Negative Binomial (r, p) random variable is the sum of r independent Geometric (p) random variables

$$\underbrace{F F F S}_{X_1} \mid \underbrace{F F F F S}_{X_2} \mid \underbrace{F S}_{X_3} \mid \underbrace{F F F F S}_{X_4}$$

$$X = 18 = X_1 + X_2 + X_3 + X_4 = 4 + 6 + 2 + 6$$