Example for Negative Binomial random variables: mass

Say X is a Negative Binomial $(r=4, p)$ random variable.

Find $P(X=18)$

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within the first 17 trials, need exactly 3 successes **AND** need 18th trial to succeed. (4th)

$$P(X=18) = \binom{17}{3} p^3 q^{14} p = \binom{17}{3} p^4 q^{14}$$

Check: powers of p and q must sum to 18 since need 18 trials altogether.

$p_X(x) =$

$$P(X=x) = \binom{x-1}{3} p^3 q^{x-1-3} p = \binom{x-1}{3} p^4 q^{x-4}$$

note: powers of p and q sum to x (good!)

$x-1$ trials → 3 successes and $x-1-3$ failures

xth trial succeed!

In general for a Negative Binomial (r,p) random variable,

$$p_X(x) = P(X=x) = \binom{x-1}{r-1} p^{r-1} q^{x-r} \cdot p = \binom{x-1}{r-1} p^r q^{x-r}$$

within 1st $x-1$ trials, need $r-1$ successes, $(x-1)-(r-1) = x-r$ failures

xth trial succeeds!

Note: If $r=1$ we just simplify have a Geometric(p) random variable, just # of trials until 1st success. Check mass:

$r=1 \rightarrow \quad p_X(x) = P(X=x) = \binom{x-1}{1-1} p^1 q^{x-1} = pq^{x-1}$ which matches mass of a Geometric.

$\binom{x-1}{0} = 1 = \frac{(x-1)!}{(x-1)!0!} = 1$