

Example: Suppose we draw cards from a deck until the 2 of spades appears for the 5th time. We replace cards and shuffle in between. Let X be the Number of draws required. Notice we have an independent collection of trials, each with the probability of success being $\frac{1}{52}$.

$$P_X(x) = P(X=x) = \binom{x-1}{4} p^4 q^{x-1-4} \cdot p = \boxed{\binom{x-1}{4} p^5 q^{x-5}}$$

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within first $x-1$ trials,
need 4 successes
and $x-1-4$ failures
 x th trial
needs to be
a success

" in this case

$$\boxed{\binom{x-1}{4} \left(\frac{1}{52}\right)^5 \left(\frac{51}{52}\right)^{x-5}}$$

Roll a die until the 10th value of 3 appears. Let X be the number of rolls required. Notice the trials (rolls) are independent, each with probability of success $p = \frac{1}{6}$.

$$P_X(x) = P(X=x) = \binom{x-1}{9} \left(\frac{1}{6}\right)^9 \left(\frac{5}{6}\right)^{x-1-9} \cdot \left(\frac{1}{6}\right) = \boxed{\binom{x-1}{9} \left(\frac{1}{6}\right)^{10} \left(\frac{5}{6}\right)^{x-10}}$$

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first $x-1$ rolls
need 9 occurrences of 3
 $x-1-9$ non-3's
 x th trial
to succeed