Expected value and variance of Negative Binomial \((r, p)\) random variables.

Think: \(X\) is Negative Binomial \((r, p)\)

then \(X = X_1 + X_2 + \ldots + X_r\) where the \(X_i\)'s are independent Geometric\((p)\) random variables.

So \[
E(X) = E(X_1 + X_2 + \ldots + X_r) \\
= E(X_1) + E(X_2) + \ldots + E(X_r) \\
= \frac{1}{p} + \frac{1}{p} + \ldots + \frac{1}{p} \\
= \left(\frac{r}{p}\right)
\]

also \[
\text{Var}(X) = \text{Var}(X_1 + X_2 + \ldots + X_r) \quad \text{since the } X_i\text{'s are independent} \\
= \text{Var}(X_1) + \ldots + \text{Var}(X_r) \\
= \left(\frac{q}{p^2}\right) + \ldots + \left(\frac{q}{p^2}\right) \\
= \left(\frac{rq}{p^2}\right)
\]

One more note: It takes at least \(r\) trials to reach the \(r\)th success

So \(X\) must be \(r\) or larger.

\[
P_X(x) = P(X = r) = \binom{r - 1}{x - 1} p^r q^{x - r} \quad x \geq r.
\]

If \(x < r\) then binomial coefficient is 0. safe to ignore \(x < r\) here.

With that in mind, you know if you have a random variable defined on \(r, r+1, r+2, \ldots\) \(\rightarrow\) no end to the potential values

your random variable might be negative binomial \((r, p)\).