Expected value and variance of Negative Binomial$(r,p)$ random variables.

Think: $X$ is Negative Binomial$(r,p)$

then $X = X_1 + X_2 + \dots + X_r$ where the $X_i$'s are independent Geometric$(p)$ random variables.

So
$$E(X) = E(X_1 + X_2 + \dots + X_r)$$
$$= E(X_1) + E(X_2) + \dots + E(X_r)$$
$$= \frac{1}{p} + \frac{1}{p} + \dots + \frac{1}{p}$$
$$= \frac{r}{p}$$

also
$$Var(X) = Var(X_1 + X_2 + \dots + X_r)$$ since the $X_i$'s are independent
$$= Var(X_1) + \dots + Var(X_r)$$
$$= \frac{q}{p^2} + \dots + \frac{q}{p^2}$$
$$= \frac{qr}{p^2}$$

One more note: It takes at least $r$ trials to reach the $r$th success, so $X$ must be $r$ or larger.

$$p_X(x) = P(X=r) = \binom{x-1}{r-1} p^r q^{x-r} \quad x \geq r.$$

if $x<r$ then binomial coefficient is 0. safe to ignore $x<r$ here.

With that in mind, you know if you have a random variable defined on $r, r+1, r+2, \dots$ ← no end to the potential values your random variable might be negative binomial$(r,p)$.