

Examples with Poisson random variables. Say that the number of cars that pass in a minute is random with an average of 2.5 cars per minute. Let X be the number of cars that pass during the next 1 minute. Find the mass of X . Here $\lambda = 2.5$, given. So we can calculate

$$p_X(x) = \frac{e^{-2.5}(2.5)^x}{x!}.$$

For instance, some specific values are:

$$p_X(0) = \frac{e^{-2.5}(2.5)^0}{0!} = 0.0821 \quad \text{this is the probability of no cars in the next minute}$$

$$p_X(1) = P(X = 1) = \frac{e^{-2.5}(2.5)^1}{1!} = 0.2052.$$

$$p_X(2) = \frac{e^{-2.5}(2.5)^2}{2!} = 0.2565.$$

$$p_X(3) = \frac{e^{-2.5}(2.5)^3}{3!} = 0.2138.$$

$$p_X(4) = \frac{e^{-2.5}(2.5)^4}{4!} = 0.1336.$$

Etc., etc. If we were to sum all of the mass values, we would have $\sum_{x=0}^{\infty} \frac{e^{-2.5}(2.5)^x}{x!} = 1$.

Same problem, suppose that there are 2.5 cars (on average) per minute. Now consider the number of cars that pass in the next 5 minutes. Let Y denote this random variable. Then Y has $\lambda = (5)(2.5) = 12.5$, because on average there are 12.5 cars that pass in the next minute. It is really, really important to make sure that we get this average correct before we do the calculations. So the probability that exactly y cars pass in the next 5 minutes is

$$p_Y(y) = P(Y = y) = \frac{e^{-12.5}(12.5)^y}{y!}.$$

Notice that X had a mass that peaked near its mean, i.e., around 2 or 3. We should expect that the mass of Y will peak about its mean, i.e., around 12 or 13. Some values of the mass of Y are:

$$p_Y(10) = \frac{e^{-12.5}(12.5)^{10}}{10!} = 0.0956.$$

$$p_Y(11) = \frac{e^{-12.5}(12.5)^{11}}{11!} = 0.1087.$$

$$p_Y(12) = \frac{e^{-12.5}(12.5)^{12}}{12!} = 0.1132.$$

$$p_Y(13) = \frac{e^{-12.5}(12.5)^{13}}{13!} = 0.1089.$$

$$p_Y(14) = \frac{e^{-12.5}(12.5)^{14}}{14!} = 0.0972.$$

It is helpful to compute some of these values yourself. Highly encouraged!