Return to example where 20 CDs, 5 desirable, 15 undesirable, pick 3 and let $X$ denote the number of desirable items among the 3.

$N = 20$, $M = 5$, $N - M = 15$

$n = 3$

$X_j = 1$ if jth item is desirable

$X_j = 0$ otherwise

$E(X^2) = E((X_1 + X_2 + X_3)(X_1 + X_2 + X_3))$

$= E(X_1 X_1) + E(X_2 X_2) + E(X_3 X_3)$

$+ E(X_2 X_1) + E(X_2 X_2) + E(X_3 X_3)$

$+ E(X_3 X_1) + E(X_3 X_2) + E(X_3 X_3)$

all $E(X_i; X_j)$ terms are the same

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Just think about changing order of selection.

$E(X^2) = 3E(X_1 X_1) + (3)(2)E(X_1 X_2)$

$= \frac{3}{6} E(X_1 X_1) + (3)(2)E(X_1 X_2)$

where $X_1$ is an indicator random variable so $X_1 X_1 = X_1^2$

So $E(X_1 X_1) = E(X_1) = \frac{5}{20} = \frac{1}{4}$

also $X_1$ and $X_2$ are indicators that are dependent on each other

$X_1 \cdot X_2 = 1$ if $X_1 = 1$ and $X_2 = 1$

$X_1 \cdot X_2 = 0$ otherwise

$E(X_1 X_2) = P(X_1 X_2 = 1) + 0P(X_1 X_2 = 0)$

$= P(X_1 = 1 \text{ and } X_2 = 1)$

$= P(X_1 = 1) P(X_2 = 1 | X_1 = 1) = \frac{1}{4} \cdot \frac{4}{19} = \frac{1}{19}$

$E(X^2) = (3)(\frac{1}{4}) + (3)(2)(\frac{1}{19})$

$= \frac{3}{4} + \frac{6}{19}$