

Discrete Uniform Random Variable

Say X is a discrete uniform random variable if X takes on values $1, 2, 3, \dots, n$, and all of these values are equally likely.

E.g. $P_X(x) = P(X=x) = \frac{1}{n}$ for $x=1, 2, \dots, n$

$$\begin{aligned} E(X) &= \frac{n+1}{2} \quad \text{Why?} & E(X) &= (1)\left(\frac{1}{n}\right) + (2)\left(\frac{1}{n}\right) + \dots + (n)\left(\frac{1}{n}\right) \\ & & &= \left(\frac{1}{n}\right)(1+2+3+\dots+n) \\ & & &= \left(\frac{1}{n}\right) \frac{n(n+1)}{2} \\ & & &= \frac{n+1}{2}. \end{aligned}$$

$$\text{Var}(X) = \frac{n^2-1}{12}.$$

Key example: X is the result on the roll of a 6-sided die
(or any sided would work)

If X is discrete uniform on $1, 2, \dots, 6$

$$\begin{aligned} \text{then } E(X) &= \frac{6+1}{2} = \frac{7}{2} \\ &= 3.5 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= \frac{6^2-1}{12} = \frac{35}{12} \\ &= 2.9167 \end{aligned}$$

We will also later have a continuous uniform random variable, and we will have much more to say about those.