

Counting: Draw 5 cards without replacement from a 52 card deck, keep track of the order of selection of the cards.

$\frac{52 \text{ ways}}{\text{card 1}}$
 $\frac{51 \text{ ways}}{\text{card 2}}$
 $\frac{50 \text{ ways}}{\text{card 3}}$
 $\frac{49 \text{ ways}}{\text{card 4}}$
 $\frac{48 \text{ ways}}{\text{card 5}}$

altogether # of ways (without replacement, keeping track of the order) to deal 5 cards out of 52 is

$$\begin{aligned}
 & \frac{(52)(51)(50)(49)(48)}{(47)(46)\dots(1)} \\
 & = \frac{(52)(51)(50)(49)(48)(47)(46)\dots(1)}{(47)(46)\dots(1)} \\
 & = \frac{52!}{47!}
 \end{aligned}$$

In general, if there are n items and we want to pick r of them without replacement, but keeping track of the order of selection,

there are

$\frac{n}{\text{item 1}}$
 $\frac{n-1}{\text{item 2}}$
 $\frac{n-2}{\text{item 3}}$
 ...
 $\frac{n-(r-1)=n-r+1}{\text{item } r}$

altogether there are

$$\begin{aligned}
 (n)(n-1)(n-2)\dots(n-r+1) &= \frac{(n)(n-1)(n-2)\dots(n-r+1)(n-r)\dots(1)}{(n-r)\dots(1)} \\
 \text{ways} &
 \end{aligned}$$

$$= \frac{n!}{(n-r)!}$$

ways to pick r out of n items without replacement, keeping track of the order of selection.