

Counting problem: Seating arrangements.

Say 5 men, 5 women sitting in a row of 10 chairs.

Say all the seating arrangements are equally likely.

Let  $X$  denote the number of couples who are sitting together.

Find  $E(X)$ . Notice I do not ask for the mass of  $X$ ,  
in deed the mass of  $X$  is challenging to compute!

Notice  $X = X_1 + X_2 + \dots + X_9$  where

$X_j$  indicates if the  $j$ th pair of chairs has  
a couple in it.

$X_j = 1$  if  $j$ th pair has a couple  
 $= 0$  otherwise.

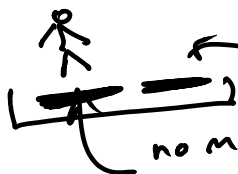


$$E(X) = E(X_1 + \dots + X_9) = E(X_1) + \dots + E(X_9)$$

↑ all nine of these are the same expectation.

Find  $E(X_j)$  <sup>some  $j$ .</sup>

$$= \mathbb{P}(X_j = 1) + 0 \mathbb{P}(X_j = 0) = \mathbb{P}(X_j = 1) = \frac{1}{9}$$



there are 9 people equally likely to sit here, and exactly  
1 is the partner of the person on the left

$$E(X) = \frac{1}{9} + \frac{1}{9} + \dots + \frac{1}{9} = (9) \left(\frac{1}{9}\right) = 1$$

So we expect exactly one couple to be sitting together.