Continuous random variables

With discrete random variable X, say $0 \le X \le 20$

If we want $P(3 \le X \le 5) = \sum_{3 \le x \le 5} P(X=x)$

e.g. if X takes on integer values

$= P(X=3) + P(X=4) + P(X=5)$

With continuous random variables, we integrate the <u>probability density function</u>, $f_X(x)$, to get probabilities. E.g. if continuous random variable X has <u>density</u> $f_X(x)$, then $P(3 \le X \le 5) = \int_3^5 f_X(x)\,dx$.

Since the random variable has to have some real value, then

$$\int_{-\infty}^{\infty} f_X(x)\,dx = P(-\infty < X < \infty) = 1.$$

Recall that the probability mass function $p_X(x)$ of a random variable X is always between 0 and 1. The values of the mass are themselves probabilities.

Not so with density values. We only require $f_X(x) \ge 0$ for all x, and $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$.

Example, say X has density $f_X(x) = 3$ for $0 \le x \le \frac{1}{3}$. So the random variable X is always between 0 and 1/3 in this case.

E.g. $P(0 \le X \le \frac{1}{6}) = \int_0^{1/6} 3\,dx = 3x\Big|_{x=0}^{1/6} = \frac{1}{2}$. So the random variable X is between 0 and 1/6 with probability 1/2.