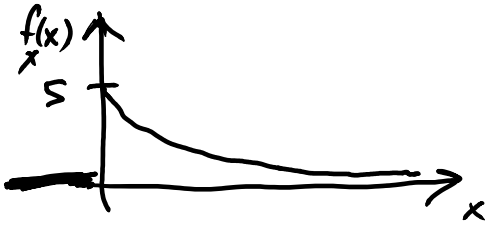


Example Say  $X$  has density  $f_X(x) = \begin{cases} 5e^{-5x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$



$$\begin{aligned} \text{Find } P(X > 2) &= \int_2^{\infty} 5e^{-5x} dx \\ &= \frac{5e^{-5x}}{-5} \Big|_{x=2}^{\infty} \\ &= e^{-10} \end{aligned}$$

$$\begin{aligned} \text{Find } P(X \leq 2) &= \int_{-\infty}^2 f_X(x) dx \\ &= \int_{-\infty}^0 f_X(x) dx + \int_0^2 f_X(x) dx \\ &= \int_{-\infty}^0 0 dx + \int_0^2 5e^{-5x} dx \\ &= \frac{5e^{-5x}}{-5} \Big|_{x=0}^2 \\ &= 1 - e^{-10} \end{aligned}$$

$$\text{Find } P(X=2) = \int_2^2 5e^{-5x} dx = \frac{5e^{-5x}}{-5} \Big|_{x=2}^2 = e^{-10} - e^{-10} = 0.$$

This is a special case of a much more general phenomenon.

For any continuous random variable  $X$  and any density  $f_X(x)$  function that defines such an  $X$ , we have  $P(X=a) = 0$  for any  $a$ . Why?

$$P(X=a) = \int_a^a f_X(x) dx = 0.$$

This is different from the behavior with discrete random variables.

It gives us some freedom in the way we write inequalities:

$$P(2 \leq X \leq 5) = \int_2^5 f_X(x) dx \quad P(2 < X < 5) = \int_2^5 f_X(x) dx$$

difference is  $P(X=2) = 0$   
 $P(X=5) = 0$

So we get the same answers either way.

Point of this:

$$P(2 \leq X \leq 5) = P(2 < X < 5) = P(2 \leq X < 5) = P(2 < X \leq 5)$$

These are all exactly the same values, all are  $\int_2^5 f_X(x) dx$ .

$P(X=2) = 0$  and  $P(X=5) = 0$  so we drop or add boundaries as needed. No worries.