Say \( X \) has density \( f_X(x) = \begin{cases} 5e^{-5x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases} \)

Find \( P(X > 2) = \int_2^\infty 5e^{-5x} \, dx \)

\[
= \frac{5e^{-5x}}{-5} \bigg|_{x=2}^{x=\infty} = e^{-10}
\]

Find \( P(X \leq 2) = \int_{-\infty}^2 f_X(x) \, dx \)

\[
= \int_{-\infty}^0 f_X(x) \, dx + \int_0^2 f_X(x) \, dx
= \int_{-\infty}^0 0 \, dx + \int_0^2 5e^{-5x} \, dx
= \frac{5e^{-5x}}{-5} \bigg|_{x=0}^{x=2} = 1 - e^{-10}.
\]

Find \( P(X=2) = \int_2^2 5e^{-5x} \, dx = \frac{5e^{-5x}}{-5} \bigg|_{x=2} = e^{-10} - e^{-10} = 0 \).

This is a special case of a much more general phenomenon. For any continuous random variable \( X \) and any density \( f_X(x) \) function that defines such an \( X \), we have \( P(X=a) = 0 \) for any \( a \). Why?

\[ P(X=a) = \int_a^a f_X(x) \, dx = 0. \]

This is different from the behavior with discrete random variables. It gives us some freedom in the way we write inequalities:

\[ P(2 \leq X \leq 5) = \int_2^5 f_X(x) \, dx \]

Point of this:

\[ P(2 \leq X \leq 5) = P(2 < X < 5) = P(2 \leq X < 5) = P(2 < X \leq 5) \]

These are all exactly the same values, all are \( \int_2^5 f_X(x) \, dx \).

\[ P(X=2) = 0 \text{ and } P(X=5) = 0 \text{ so we respect or add boundaries as needed. No worries.} \]