Cumulative distribution function \[ F_X(x) = P(X \leq x) \]
\[ F_X(a) = P(X \leq a) \]

With continuous random variables, \[ F_X(a) = P(X \leq a) = \int_{-\infty}^{a} f_X(x) dx \]

For example, if \( f_X(x) = 3 \) for \( 0 \leq x \leq \frac{1}{3} \)
\[ = 0 \text{ otherwise} \]
\[ F_X(a) = P(X \leq a) = 0 \text{ for } a < 0. \quad \text{Why: } \int_{-\infty}^{a} f_X(x) dx = \int_{-\infty}^{0} 0 dx = 0 \]
\[ F_X(a) = P(X \leq a) = 1 \text{ for } a > \frac{1}{3}. \quad \text{Why: } \int_{-\infty}^{a} f_X(x) dx \]
\[ = \int_{-\infty}^{0} 3 dx + \int_{0}^{a} 3 dx 
\[ = 0 + 1 + 0 \]

For "a" in the interesting region: \( 0 \leq a \leq \frac{1}{3} \)
\[ F_X(a) = P(X \leq a) = \int_{-\infty}^{a} f_X(x) dx = \int_{-\infty}^{0} 0 dx + \int_{0}^{a} 3 dx 
\[ = 0 + 3a \]

What is the graph of the CDF? \( F_X(a) \)

Other example: Say \( X \) has density \( f_X(x) = \begin{cases} \frac{5}{x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases} \)
\[ f_X(x) \]
\[ \int_{-\infty}^{a} f_X(x) dx \]

How does the CDF look? CDF \( F_X(a) = \int_{-\infty}^{a} f_X(x) dx = \int_{-\infty}^{0} 0 dx = 0 \)
\[ F_X(a) = \int_{-\infty}^{a} f_X(x) dx \]
\[ = \int_{-\infty}^{0} 0 dx + \int_{0}^{a} \frac{5}{x} dx 
\[ = 0 + \frac{5}{x} \bigg|_{x=0}^{a} 
\[ = \frac{1 - e^{-5a}}{1 - e^{-5}} \]