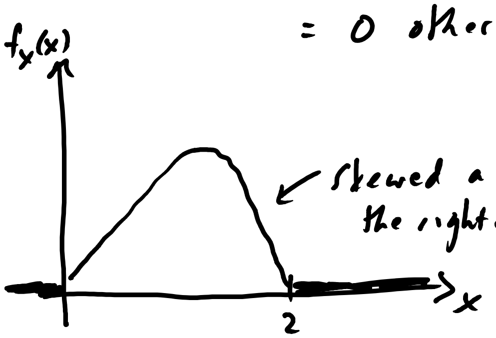
Example Say X is a continuous random variable with density

$$f_X(x) = \frac{x}{4}(2-x)(2+x) \quad \text{for } 0<x<2$$
$$= 0 \text{ otherwise}$$

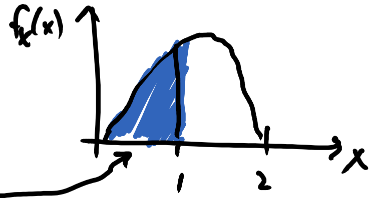


skewed a little to the right.

Find $P(X\le 1) = \int_{-\infty}^{1} f_X(x)\,dx$

$$= \cancel{\int_{-\infty}^{0} 0\,dx} + \int_0^1 \frac{x}{4}(2-x)(2+x)\,dx$$
$$= \int_0^1 \frac{1}{4}(4x - x^3)\,dx$$
$$= \frac{1}{4}\left(\frac{4x^2}{2} - \frac{x^4}{4}\right)\Big|_{x=0}^{1}$$
$$= \frac{1}{4}\left(2 - \frac{1}{4}\right) = \frac{1}{4}\left(\frac{7}{4}\right) = \frac{7}{16}$$

$P(X\le 1) = \frac{7}{16}$



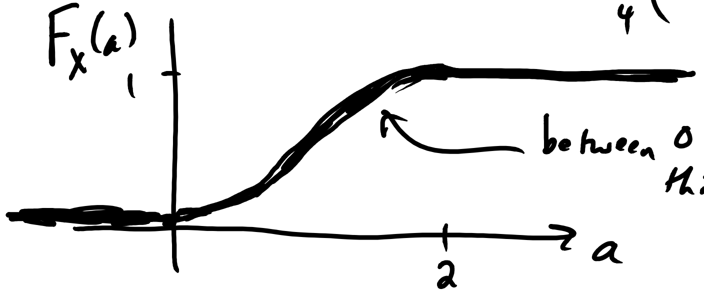
Same as the area under the curve of the density from $x=0$ to $x=1$.

Now find CDF of X: $F_X(a) = P(X\le a)$

For $a<0$, $F_X(a) = 0$
$a>2$, $F_X(a) = 1$
} non interesting regions

For $0<a<2$:

$$F_X(a) = P(X\le a) = \int_{-\infty}^{a} f_X(x)\,dx = \cancel{\int_{-\infty}^{0} 0\,dx} + \int_0^a \frac{x}{4}(2-x)(2+x)\,dx$$
$$= \int_0^a \frac{1}{4}(4x - x^3)\,dx$$
$$= \frac{1}{4}\left(\frac{4x^2}{2} - \frac{x^4}{4}\right)\Big|_{x=0}^{a} = \frac{1}{4}\left(2a^2 - \frac{a^4}{4}\right) = \frac{a^2}{2} - \frac{a^4}{16}$$



between 0 and 2, this $\frac{a^2}{2} - \frac{a^4}{16}$

Double check: If we integrate the CDF $F_X(x)$ for $0<x<2$, do we get the density back?

$$\frac{d}{dx}F_X(x) = \frac{d}{dx}\left(\frac{x^2}{2} - \frac{x^4}{16}\right) = \frac{2x}{2} - \frac{4x^3}{16} = x - \frac{1}{4}x^3 = \frac{1}{4}(4x - x^3)$$
$$= \frac{x}{4}(4-x^2)$$
$$= \frac{x}{4}(2-x)(2+x)$$
$$= f_X(x) \ ✓✓$$