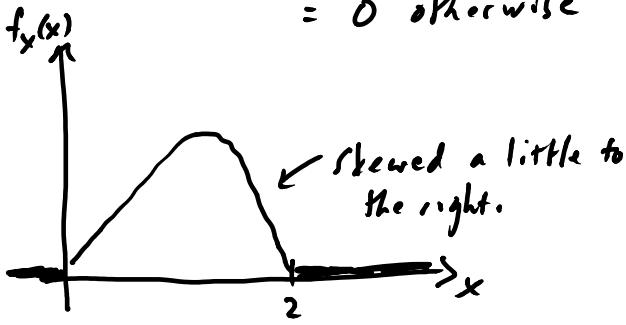


Example Say X is a continuous random variable with density

$$f_x(x) = \frac{x}{4}(2-x)(2+x) \text{ for } 0 < x < 2$$

$$= 0 \text{ otherwise}$$



Find $P(X \leq 1) = \int_{-\infty}^1 f_x(x) dx$

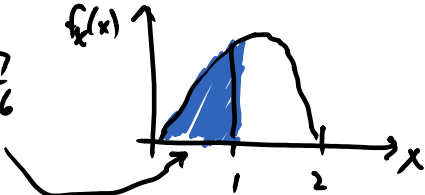
$$= \int_{-\infty}^0 0 dx + \int_0^1 \frac{x}{4}(2-x)(2+x) dx$$

$$= \int_0^1 \frac{1}{4}(4x - x^3) dx$$

$$= \frac{1}{4} \left(\frac{4x^2}{2} - \frac{x^4}{4} \right) \Big|_{x=0}^1$$

$$= \frac{1}{4} \left(2 - \frac{1}{4} \right) = \frac{1}{4} \left(\frac{7}{4} \right) = \frac{7}{16}$$

$$P(X \leq 1) = \frac{7}{16}$$



Same as the area under the curve of the density from $x=0$ to $x=1$.

Now find CDF of X : $F_x(a) = P(X \leq a)$

For $a \leq 0$, $F_x(a) = 0$

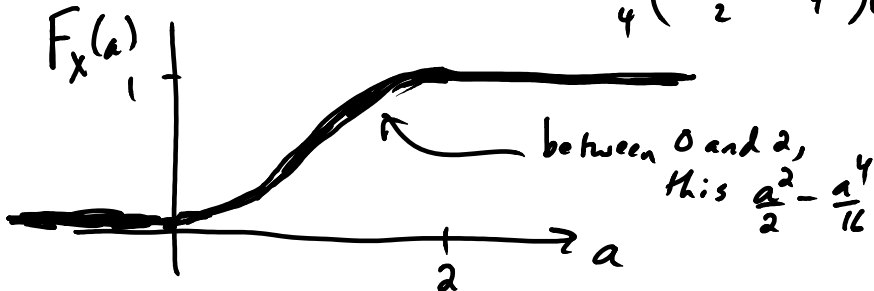
$a > 2$, $F_x(a) = 1$ } non interesting regions

For $0 < a < 2$:

$$F_x(a) = P(X \leq a) = \int_{-\infty}^a f_x(x) dx = \int_{-\infty}^0 0 dx + \int_0^a \frac{x}{4}(2-x)(2+x) dx$$

$$= \int_0^a \frac{1}{4}(4x - x^3) dx$$

$$= \frac{1}{4} \left(\frac{4x^2}{2} - \frac{x^4}{4} \right) \Big|_{x=0}^a = \frac{1}{4} \left(2a^2 - \frac{a^4}{4} \right) = \frac{a^2}{2} - \frac{a^4}{16}$$



Double check: If we integrate the CDF $F_x(x)$ for $0 < x < 2$, do we get the density back?

$$\frac{d}{dx} F_x(x) = \frac{d}{dx} \left(\frac{x^2}{2} - \frac{x^4}{16} \right) = \frac{2x}{2} - \frac{4x^3}{16} = x - \frac{1}{4}x^3 = \frac{1}{4}(4x - x^3)$$

$$= \frac{x}{4}(4 - x^2)$$

$$= \frac{x}{4}(2-x)(2+x)$$

$$= f_x(x) \checkmark \checkmark$$