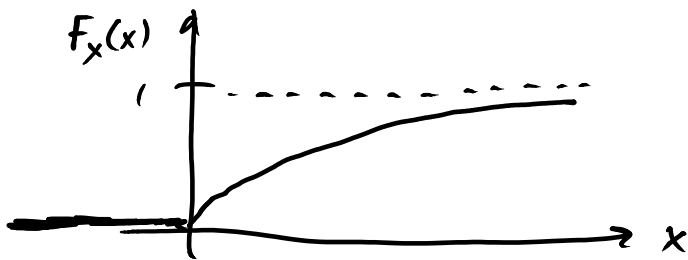


Example: Say X has CDF $F_X(x) = 1 - e^{-x/4}$ for $x > 0$
 $= 0$ otherwise



What is the density of X ?

$$f_X(x) = 0 \text{ for } x < 0$$

and for $x \geq 0$, we have

$$\begin{aligned} f_X(x) &= \frac{d}{dx} (F_X(x)) \\ &= \frac{d}{dx} (1 - e^{-x/4}) \\ &= 0 - (e^{-x/4} \cdot \left(-\frac{1}{4}\right)) \\ &= \frac{1}{4} e^{-x/4} \end{aligned}$$

Check that this is a valid density:

$$f_X(x) = \begin{cases} \frac{1}{4} e^{-x/4} & x > 0 \\ 0 & \text{otherwise} \end{cases} \text{ always positive or 0 (never negative)}$$

$$\int_{-\infty}^{\infty} f_X(x) dx = \int_{-\infty}^0 0 dx + \int_0^{\infty} \frac{1}{4} e^{-x/4} dx = \frac{1}{4} \frac{e^{-x/4}}{-1/4} \Big|_{x=0}^{\infty} = 1. \checkmark$$

Started with the CDF, obtained the density, by differentiating.