**Example** Suppose \( X \) and \( Y \) have joint density \( f_{X,Y}(x,y) = \begin{cases} 6e^{-2x-3y} & x > y \\ 0 & \text{otherwise} \end{cases} \)

So in particular, \( X \) and \( Y \) are both positive.

Find \( P(X > Y) = \iint_{x>y} f_{X,Y}(x,y) \, dx \, dy \)

\[
\begin{align*}
&= \int_0^\infty \left[ \int_y^\infty 6e^{-2x-3y} \, dx \right] \, dy \\
&= \int_0^\infty \left[ -3e^{-2x-3y} \right]_y^\infty \, dy \\
&= \int_0^\infty 3e^{-5y} \, dy \\
&= \left[ \frac{3}{5} e^{-5y} \right]_0^\infty \\
&= \frac{3}{5}
\end{align*}
\]

So \( P(X > Y) = \frac{3}{5} \).

What about \( P(X = Y) \)? Must be 0!

\[
P(X > Y) + P(X = Y) + P(X < Y) = 1
\]

\[
\frac{3}{5} + 0 + \frac{2}{5} = 1
\]

Check directly:

\[
P(X = Y) = \int_0^\infty \int_x^\infty 6e^{-2x-3y} \, dy \, dx = \int_0^\infty 0 \, dx = 0.
\]

This is a common phenomenon for continuous random variables. The probability of exact ties is often 0.