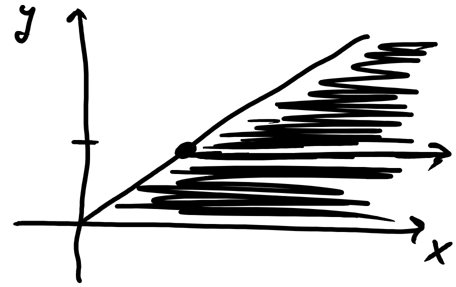
<u>Example</u> Suppose X and Y have joint density $f_{X,Y}(x,y) = \begin{cases} 6e^{-2x-3y} & x>0, y>0 \\ 0 & \text{otherwise} \end{cases}$

So in particular, X and Y are both positive.

Find $P(X>Y) = \iint\limits_{x>y} f_{X,Y}(x,y)\, \underbrace{dy\, dx}_{\text{or } dx\, dy}$

haven't yet determined how to specify the boundaries



$$= \int_0^\infty \boxed{\int_y^\infty 6e^{-2x-3y}\, dx}\, dy$$

$$= \int_0^\infty \boxed{\left.\frac{6e^{-2x-3y}}{-2}\right|_{x=y}^{\infty}}\, dy$$

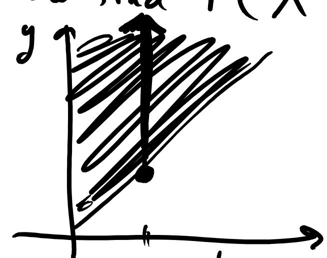
$$= \int_0^\infty 3e^{-2y-3y}\, dy$$

$$= \left.\frac{3e^{-5y}}{-5}\right|_{y=0}^{\infty}$$

$$= \boxed{\frac{3}{5}}$$

So $\underline{P(X>Y) = 3/5}$.

Now find $P(X<Y)$



$$P(X<Y) = \int_0^\infty \boxed{\int_x^\infty 6e^{-2x-3y}\, dy}\, dx$$

$$= \int_0^\infty \boxed{\left.\frac{6e^{-2x-3y}}{-3}\right|_{y=x}^{\infty}}\, dx$$

$$= \int_0^\infty 2e^{-2x-3x}\, dx$$

$$= \left.\frac{2e^{-5x}}{-5}\right|_{x=0}^{\infty}$$

$$= \boxed{\frac{2}{5}}$$

So $\underline{P(X<Y) = \frac{2}{5}}$

Q: $P(X=Y)$??? Must be 0!

$$P(X>Y) + P(X=Y) + P(X<Y) = 1$$

$$3/5 + 0 + 2/5 = 1$$

Check directly:

$$P(X=Y) = \int_0^\infty \boxed{\int_x^x 6e^{-2x-3y}\, dy}\, dx = \int_0^\infty 0\, dx = 0.$$

This is a common phenomena for continuous random variables.

The probability of exact ties is often 0.