

Use the joint density of  $X$  and  $Y$  to find the single variable density of  $X$  or of  $Y$ . (Sometimes called the marginal density of  $X$  or  $Y$ .)

In general if  $X$  and  $Y$  have joint density  $f_{X,Y}(x,y)$

then  $X$  has density  $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy$  i.e. think: integrating  $y$  out of the picture.

and  $Y$  has density  $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx$  i.e. integrate  $x$  out of the picture.

Example Say  $X, Y$  have joint density  $f_{X,Y}(x,y) = \begin{cases} 6e^{-2x-3y} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$

Then  $X$  has density  $f_X(x) = 0$  if  $x < 0$

$$\begin{aligned} \text{or if } x > 0: f_X(x) &= \int_0^{\infty} 6e^{-2x-3y} dy = \frac{6e^{-2x-3y}}{-3} \Big|_{y=0}^{\infty} \\ &= 2e^{-2x} \text{ (for } x > 0) \end{aligned}$$

Also  $Y$  has density:  $f_Y(y) = 0$  if  $y < 0$

$$\begin{aligned} \text{or if } y > 0: f_Y(y) &= \int_0^{\infty} 6e^{-2x-3y} dx = \frac{6e^{-2x-3y}}{-2} \Big|_{x=0}^{\infty} = 3e^{-3y} \\ &\text{(for } y > 0) \end{aligned}$$

This method works in general if you have the joint density of  $X$  and  $Y$  but just want the single variable density of  $X$  or of  $Y$ .