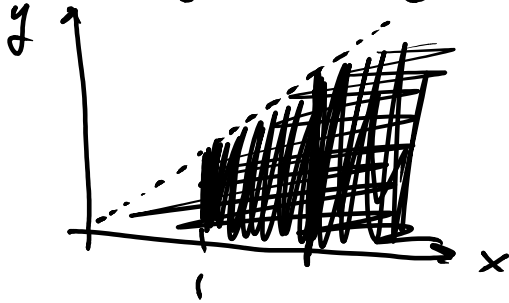


One more example: Suppose X, Y have joint density

$$f_{X,Y}(x,y) = \begin{cases} 9e^{-3x} & \text{when } 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

Notice the joint density is defined here:



Find $P(X > 1)$.

Two methods:

$$P(X > 1) = \int_1^{\infty} \int_0^x 9e^{-3y} dy dx$$

$$= \int_1^{\infty} 9e^{-3x} \cdot y \Big|_{y=0}^x dx$$

$$= \int_1^{\infty} 9xe^{-3x} dx$$

$$u = 9x$$

$$dv = e^{-3x} dx$$

use int. by parts

$$du = 9 dx$$

$$v = \frac{e^{-3x}}{-3}$$

$$= (9x) \left(\frac{e^{-3x}}{-3} \right) \Big|_{x=1}^{\infty} - \int_1^{\infty} (9) \left(\frac{e^{-3x}}{-3} \right) dx$$

$$= 9 \cdot 1 \cdot \frac{e^{-3}}{3} + \int_1^{\infty} 3e^{-3x} dx$$

$$= 3e^{-3} + \frac{3e^{-3x}}{-3} \Big|_{x=1}^{\infty}$$

$$= 3e^{-3} + e^{-3}$$

$$= 4e^{-3} \approx .199 \quad \text{i.e. about 20\% of the time, } X \text{ is larger than 1.}$$

Method 2 First get $f_X(x)$.

$$\begin{aligned} f_X(x) &= \int_0^x 9e^{-3y} dy \\ &= 9e^{-3x} \cdot y \Big|_{y=0}^x \\ &= 9xe^{-3x} \end{aligned}$$

Now I know density of X is $f_X(x) = \begin{cases} 9xe^{-3x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

$$P(X > 1) = \int_1^{\infty} 9xe^{-3x} dx = (9x) \left(\frac{e^{-3x}}{-3} \right) \Big|_{x=1}^{\infty} - \int_1^{\infty} 9 \frac{e^{-3x}}{-3} dx$$

Need u-sub

$$u = 9x$$

$$du = 9 dx$$

$$dv = e^{-3x} dx$$

$$v = \frac{e^{-3x}}{-3}$$

= ... = exactly same steps = $4e^{-3} \approx .199$.

You can use whichever method makes you more comfortable.