Example of two continuous random variables that are independent. Say X and Y have joint density:

$$f_{X,Y}(x, y) = \frac{1}{150}(8 - x^3)(5 - y) \qquad \text{for } 0 \leq x \leq 2, \text{ and } 0 \leq y \leq 5.$$

First, we can check that this is a valid joint density, e.g.,

$$\int_0^2 \int_0^5 f_{X,Y}(x, y)\, dy\, dx = 1.$$

Since the joint density $f_{X,Y}(x, y)$ can be factored in such a way that the factoring works for all x, y, then we know that X and Y are independent. In fact, the density of X must be some multiple of $(8 - x^3)$ and the density of Y must be some multiple of $5 - y$. Let's check:

$$\int_0^2 (8 - x^3)\, dx = (8x - x^4/4)|_{x=0}^2 = 16 - 4 = 12.$$

Therefore, if we divide by 12 throughout the equation, we have

$$\int_0^2 \frac{1}{12}(8 - x^3)\, dx = 1.$$

So we claim that $f_X(x) = (1/12)(8 - x^3)$ for $0 \leq x \leq 2$.

Now we check the density of Y. We claim that it is a multiple of $5 - y$. We calculate:

$$\int_0^5 (5 - y)\, dy = (5y - y^2/2)|_{y=0}^5 = 25 - 25/2 = 25/2.$$

Therefore, if we multiple throughout by $2/25$ (i.e., divide throughout by $25/2$) we get

$$\int_0^5 (2/25)(5 - y)\, dy = 1.$$

Finally, we have

$$(1/150)(8 - x^3)(5 - y) = (1/12)(8 - x^3)(2/25)(5 - y),$$

for $0 \leq x \leq 2$ and $0 \leq y \leq 5$. So X and Y are independent, and we also found the densities of X and Y individually, from the joint density, i.e., $f_X(x) = (1/12)(8 - x^3)$ for $0 \leq x \leq 2$, and $f_Y(y) = (2/25)(5 - y)$ for $0 \leq y \leq 5$.