Example: Say X and Y are independent random variables with densities: $f_X(x) = (3/8)e^{-(3/8)x}$ for $x > 0$, and $f_X(x) = 0$ otherwise. Similarly, say $f_Y(y) = (3/8)e^{-(3/8)y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Let $Z = \min\{X, Y\}$. Find $P(Z \leq 1)$.

Since X, Y are independent, then their joint density is just the product of their densities. So

$$f_{X,Y}(x, y) = (3/8)e^{-(3/8)x}(3/8)e^{-(3/8)y},$$

for $x > 0$ and $y > 0$, and $f_{X,Y}(x, y) = 0$ otherwise.

Now let's find the density of Z. It is easier to find the CDF of Z. Since Z is the minimum of X and Y, then $Z > a$ if and only if $X > a$ and $Y > a$. So

$$P(Z > a) = P(X > a)P(Y > a) = (\int_a^\infty (3/8)e^{-(3/8)x}\,dx)(\int_a^\infty (3/8)e^{-(3/8)y}\,dy).$$

We calculate:

$$\int_a^\infty (3/8)e^{-(3/8)x}\,dx = -e^{-(3/8)x}|_{x=a}^{\infty} = e^{-(3/8)a}.$$

So altogether we have

$$P(Z > a) = (e^{-(3/8)a})(e^{-(3/8)a}) = e^{-(3/4)a}.$$

So the CDF of Z is:

$$F_Z(a) = 1 - e^{-(3/4)a}, \qquad \text{for } a > 0.$$

So the density of Z, in particular, is $f_Z(z) = (3/4)e^{-(3/4)z}$.

Now we have two ways to compute $P(Z \leq 1)$.

One way: $P(Z \leq 1) = F_Z(1) = 1 - e^{-3/4} \approx 0.5276$.

Or another way is to just integrate (if you did not happen to notice that you could plug in to the CDF), we could calculate $P(Z \leq 1) = \int_0^1 (3/4)e^{-(3/4)z}\,dz = -e^{-(3/4)z}|_{z=0}^{1} = 1 - e^{-3/4} \approx 0.5276$.