

Conditional density. The conditional density of X , given that $Y = y$, is only defined for values y such that $f_Y(y) > 0$. For such y , we define the conditional density of X , given $Y = y$, as

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}.$$

Notice that this equivalent to have

$$f_{X,Y}(x,y) = f_Y(y)f_{X|Y}(x|y).$$

In many ways this parallels how conditional masses were defined for discrete random variables. The idea of the conditional density of X , given $Y = y$, is to understand how X behaves, knowing that $Y = y$.