Expected values of continuous random variables. If X is a continuous random variable, with density $f_X(x)$, we integrate x times the density to get the expected value of X. In other words,

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx.$$

This is very much like how we sum x times the probability mass function of a discrete random variable to get the expected value of the random variable. As with discrete random variables, where we often ignore the values of the mass function that are 0, we can do the same with continuous random variables. For instance, if we have a continuous random variable X that is always non-negative, then it is unnecessary to integrate the part from $-\infty$ to 0. Why?

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx = \int_{-\infty}^{0} x f_X(x)\, dx + \int_{0}^{\infty} x f_X(x)\, dx.$$

If the density of X is just 0 for $x < 0$, i.e., if $f_X(x) = 0$ for $x < 0$, then this simplifies to:

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx = \int_{0}^{\infty} x f_X(x)\, dx.$$

Nothing special about that example. For example, suppose that $f_X(x) > 0$ for $0 < x < 10$ and $f_X(x) = 0$ otherwise. Then

$$E(X) = \int_{0}^{10} x f_X(x)\, dx$$

because the density is just 0 the rest of the time, i.e., for $x > 10$ and for $x < 0$.

The main point is that, to find the expected value of X, we integrate x times the density $f_X(x)$ over the region where $f_X(x) > 0$. It is OK to integrate over further x's too, but the rest of the integral will just give us 0.