

Example: Suppose that the density of X is $f_X(x) = (3/125)x^2$ for $0 < x < 5$, and $f_X(x) = 0$ otherwise. First, let's check to see that this is a valid density function:

$$\int_0^5 f_X(x) dx = \int_0^5 (3/125)x^2 dx = (3/125)x^3/3|_{x=0}^5 = (3/125)(5^3/3) = 1.$$

Since the density is nonnegative for all x , and since the density integrates to 1, we have a valid probability density function (pdf).

Now let's find the expected value of X . We compute:

$$E(X) = \int_0^5 x(3/125)x^2 dx = 3/125 \int_0^5 x^3 dx = (3/125)x^4/4|_{x=0}^5 = (3/125)(5^4/4) = 15/4.$$