

Sanity check with respect to expected value. Sometimes it is case that there are upper and lower bounds, say M and m , so that $m \leq X \leq M$ all the time. Then we compute the expected value of X , it is enough to just integrate from m to M . So

$$E(X) = \int_m^M x f_X(x) dx \leq \int_m^M M f_X(x) dx = M \int_m^M f_X(x) dx = M(1) = M.$$

So $E(X) \leq M$ in such a case.

Similarly, with the lower bound

$$E(X) = \int_m^M x f_X(x) dx \geq \int_m^M m f_X(x) dx = m \int_m^M f_X(x) dx = m(1) = m.$$

So $E(X) \geq m$ in such a case.

So, in summary, if we have continuous random variable X with $m \leq X \leq M$, then $m \leq E(X) \leq M$ too.