Another example, suppose $X$ has constant density on the range $[0,5]$. Since the integral of the density must be 1, this constant has to be $1/5$. In other words,

$$\int_0^5 f_X(x) \, dx = \int_0^5 \frac{1}{5} \, dx = 1.$$ 

What is the expected value of $X$?

$$E(X) = \int_0^5 x(1/5) \, dx = (1/5)(x^2/2)|_0^5 = (1/5)(5^2/2) = 5/2.$$ 

This is just a special case of a more general idea: Suppose instead that $X$ has a constant density on the range $[a,b]$. Then

$$\int_a^b f_X(x) \, dx = \int_a^b \frac{1}{b-a} \, dx = 1.$$ 

So the constant of the density has to be $1/(b-a)$. No other constant will work. So $f_X(x) = 1/(b-a)$ for $a < x < b$, and otherwise $f_X(x) = 0$. What is the expected value of such a random variable?

$$E(X) = \int_a^b (x)(1/(b-a)) \, dx = (1/(b-a))(x^2/2)|_a^b = (1/(b-a))(b^2-a^2)/2 = (1/(b-a))(b-a)(b+a)/2$$

So altogether we just get

$$E(X) = (b + a)/2.$$ 

FYI, in our example above, at the top, we got $E(X) = 5/2$, and this matches with $(5 + 0)/2$ in the special case with $a = 0$ and $b = 5$. This works much more generally than that too.