

Another example: Suppose X and Y have joint density $f_{X,Y}(x,y) = 35e^{-5x-7y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x,y) = 0$ otherwise. What is the expected value of X in this case?

Several different ways to see this. One way is to notice that X and Y must be independent, because the joint density factors cleanly into some x parts times some y parts, e.g., $35e^{-5x-7y} = (5e^{-5x})(7e^{-7y})$. So it must be the case that X has density $f_X(x) = 5e^{-5x}$ for $x > 0$ and $f_X(x) = 0$ otherwise. So using the same method from our earlier example in this set of modules, we get $E(X) = 1/5$.

Here's another method. Suppose we didn't recognize that X and Y are independent, and we manually extracted the density of X from the joint density. Remember that we do this by integrating y out of the picture:

$$f_X(x) = \int_0^\infty f_{X,Y}(x,y) dy = \int_0^\infty 35e^{-5x-7y} dy = \frac{35e^{-5x-7y}}{-7} \Big|_{y=0}^\infty = 5e^{-5x}.$$

Then (afterwards) we would just proceed as we did before to get $E(X) = 1/5$.

Here's one more way. We explore this more in the next set of modules. We have

$$E(g(X,Y)) = \int_0^\infty \int_{-\infty}^\infty g(x,y) f_{X,Y}(x,y) dy dx$$

We try it here. Let's define $g(X,Y) = X$. We have

$$E(X) = \int_0^\infty \int_0^\infty (x)(35e^{-5x-7y}) dy dx$$

The inner integral is just

$$x \int_0^\infty 35e^{-5x-7y} dy = (x) \frac{35e^{-5x-7y}}{-7} \Big|_{y=0}^\infty = (x)5e^{-5x}$$

So the entire expectation becomes

$$E(X) = \int_0^\infty (x)5e^{-5x} dx = 1/5.$$

One comment about why I fixed the range of integration earlier. In general we want to integrate from $-\infty$ to ∞ , but in this specific case, our densities are 0 for $x < 0$ and for $y < 0$, so no need to integrate over those regions. We only need to integrate over the range where the density is strictly positive. We integrate over more of the range but we just get lots and lots of 0's.