

Example: Let's consider a continuous random variable X with density function $f_X(x) = 1/5$ for $0 < x < 5$, and $f_X(x) = 0$ otherwise. We already saw that $E(X) = 5/2$. Now let's compute $E(X^2)$. We have

$$E(X^2) = \int_0^5 x^2(1/5) dx = (1/5)x^3/3|_{x=0}^5 = (1/5)5^3/3 = 25/3.$$

So we also now basically have the variance as well; just need to write

$$\text{Var}(X) = E(X^2) - (E(X))^2 = 25/3 - (5/2)^2 = (100 - 75)/12 = 25/12.$$